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NUMERICAL STUDIES OF A DETONATION ANALOGUE

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Abstract

As an aid to deciding a computational strategy for problems involving strong detonation waves, we have applied a variety of numerical techniques to a mathematical problem devised by Fickett, which exhibits many of the essential computational difficulties and possesses analytical solution corresponding to overdriven and underdriven reacting flows. Classical shock capturing methods such as those of MacCormack and Godunov, do not produce acceptable solutions. Better solutions can be provided by Flux Difference Methods (Roe's method) and by Random Choice Methods (still in one dimension), especially by using a new variant that yields second order accuracy. Adaptive gridding techniques for these methods are currently being investigated with encouraging preliminary results.

1. INTRODUCTION

Theoretical modelling of detonation phenomena in condensed media is an area of increasing interest at the present time. Proposed mathematical models usually are systems of hyperbolic partial differential equations together with appropriate boundary and initial conditions as well as correlations. The numerical solution of such equations is not a trivial problem, although great advances have been made in recent years. The main numerical difficulties arise when representing discontinuities (eg. shocks, interface). Modern methods are able to capture discontinuous features of the flow, without applying any special features to them, and yet resolving them within two to three mesh intervals. These methods are very satisfactory when applied to a chemically inactive gas, but are not wholly adequate when the wave motion is coupled with strong reactions, as in a detonation wave. The situation we wish to model involves a hydrodynamic shock of negligible thickness which triggers and is followed by a reaction zone whose thickness is small but significant. In realistic computations involving an affordable number of mesh points, the reaction zone might occupy five to ten mesh intervals, and if two or three of these have to take care of the shock, the reaction zone is not well represented.

A realistic mathematical model involves the unsteady Euler equations in two or three dimensions as basic component whose numerical solution requires a significant computational effort.

Hence in deciding an adequate numerical strategy it is best to choose an analogous mathematical problem that contains some of the basic numerical difficulties posed by realistic models. Here we use a detonation analogue proposed by Fickett[1], which has two fundamental features, namely a hydrodynamic shock and a reaction zone of finite width. In addition the analogue has exact solutions which are used to assess the adequacy of candidate numerical methods.

So far we have tested the following numerical techniques: (1) MacCormack's version of the Lax-Wendroff Scheme, (2) a Moving Finite Element Method, (3) Godunov's Method, (4) the Random Choice Method, (5) a Higher Order Random Choice Method and (6) a Flux Difference Splitting Method. It is found that methods 1 to 3 are inadequate; methods 4 and 5 perform very well, especially the latter. These methods have the capability of producing shocks (and contacts) of zero width. Multidimensional versions of these methods however, have not yet been developed to a satisfactory level. We are currently working on this problem. Method 6 performs very satisfactorily, although discontinuities are smeared; this method extends readily to multidimensional problems via space operator splitting.

In our experience, the choice of the numerical methods to use in the real models has to be coupled with adaptive gridding techniques. These will allow us to optimise the computing resources by applying finer grids where strictly required (eg.

shock and reaction zone) only. We are currently incorporating this technique into the development of numerical methods.

The remaining part of this paper is organised as follows: in Section 2 we briefly describe the detonation analogue and its likely solutions. In Section 3 we describe the main features of the numerical methods used and give references for further details. In Section 4 we describe two test problems. In Section 5 we present numerical results and the performance of the methods. In Section 6 we draw some conclusions.

2. THE DETONATION ANALOGUE

The construction of the detonation analogue considered here is due to Fickett[1]. The philosophy behind such construction is the need for a mathematical object that retains some of the essential features of reactive flow whilst avoiding many of the complexities present in mathematical models for the actual physical problem (the physical system). The proposed analogue is

$$\rho_t + p_x = 0 \tag{1}$$

$$\lambda_t = r \tag{2}$$

$$p = p(\rho,\lambda) \tag{3}$$

$$r = r(\rho,\lambda) \tag{4}$$

This is a 2 x 2 system (1) - (2) for the unknowns ρ and λ. The conserved variable ρ and the flux p are to be associated with the density and pressure of the physical system and so the same names will be used. The auxilliary relations (3) and (4) play the roles of equation of state and rate equation respectively.

A single chemical reaction A → B is allowed, with the mass fraction of B denoted by λ. Concerning p and r in equations (3) - (4), various choices are possible. Here we take

$$p = \tfrac{1}{2}(\rho^2 + \lambda Q) \tag{5}$$

$$r = 2(1 - \lambda)^{\frac{1}{2}} \tag{6}$$

where the positive constant Q plays the role of heat of reaction. The rate r is restricted to positive values only except in the equilibrium state $\lambda = 1$ where $r = 0$. When $Q = 0$ the detonation analogue reduces to the inviscid Burger's equation (the unreactive flow analogue).

Likely solutions of the analogue are depicted in Fig. 1. Fig. 1(a) illustrates the limiting case of an overdriven detonation with $\rho_b = \rho_e = S$. Here ρ_b is the left boundary function (constant in time in this case), ρ_e is the final equilibrium value (complete reaction, $\lambda = 1$) and S is the leading shock speed. Fig 1.(b) illustrates an overdriven detonation with $\rho_b = \rho_e > S$ and Fig. 1(b) illustrates an unsupported detonation with an unsteady 'following flow' region. Fig. 1(d) depicts the behaviour of the reaction progress variable λ.

For the steady state case (eg. Fig. 1(a)) one can obtain an exact solution by transforming the equations to a frame moving with the shock of constant speed S. The result is

$$\rho = S + [S^2 - Q(2 - t)t]^{\frac{1}{2}} \tag{7}$$

with $0 \leqslant t \leqslant 1$. The solution is valid for the leading shock and the reaction zone attached to it, ie. from $\lambda = 0$ ($t = 0$) to

$\lambda = 1$ ($t = 1$). Note that reaction is completed in unit time. The exact solution (7) will be used to assess the performance of the various candidate numerical methods.

3. NUMERICAL METHODS USED

We consider six numerical methods. Here we give a brief description of each of them together with appropriate references where further details can be found. In general, we want to solve a hyperbolic system of the form

$$U_t + F(U)_x = S(U) \tag{8}$$

which for the analogue discussed in Section 1 one has

$$U = \begin{bmatrix} \rho \\ \lambda \end{bmatrix}, \quad F(U) = \begin{bmatrix} \frac{1}{2}(\rho^2 + \lambda Q) \\ 0 \end{bmatrix}, \quad S(U) = \begin{bmatrix} 0 \\ r(\lambda) \end{bmatrix} \tag{9}$$

the unknowns of the problem are ρ and λ. They depend on space x and time t.

3.1 The MacCormack Method

This is a two-step version of the Lax-Wendroff finite difference method. The first step of the method (predictive) obtains a provisional value U* at an intermediate time t* by, say, forward flux differencing system (8), ie.

$$U_i^* = U_i^n - \frac{\Delta T}{\Delta X}\left[F_{i+1}^n - F_i^n\right] \tag{10a}$$

Here source terms have been ignored. The second (corrective) step advances the solution to the complete time level n+1 by differencing in the opposite direction and using predicted values U* for flux evaluation, ie.

$$U_i^{n+1} = \tfrac{1}{2}(U_i^n + U_i^*) - \frac{\Delta T/2}{\Delta X}\left[F_i^* - F_{i-1}^*\right] \qquad (10b)$$

Application of this scheme to practical situations will normally involve the use of artificial viscosity, an aspect which is not considered here. Further details of the method can be found in Ref 2.

3.2 A Moving Finite Element Method

The moving finite element (MFE) method is a relatively new technique still under active development. It can be explained by reference to the scalar hyperbolic equation

$$U_t + f_x = 0 \qquad (11)$$

where to begin with we consider only the homogeneous (source-free) problem.

An approximation is sought by replacing U by a piecewise linear function V of the form

$$V = \Sigma_i\, a_i \alpha_i \qquad (12)$$

where

$a_i = a_i(t)$, $i = 1, \ldots, i_{max}$, are nodal amplitudes

$\alpha_i = \alpha_i(\underline{X},\underline{S})$, $i = 1, \ldots, i_{max}$, are linear basis functions of local compact support and $\underline{S} = \underline{S}(t)$ is a time-dependent vector of nodal positions S_i , $i = 1, \ldots, i_{max}$.

Partial differential of (11) with respect to time yields

$$V_t = \Sigma_i(\dot{a}_i\alpha_i + \dot{S}_i\beta_i) \qquad (13)$$

where

α_i and β_i are basis functions defined by the same grid points (locally).

The equation (11) is replaced by

$$V_t + f(V)_x = 0 \qquad (14)$$

where V_t is given in (13). The flux is projected into the approximation space spanned by the basis functions α_i and β_i by minimising the L_2 residual

$$\left|\left| V_t + f(V)_x \right|\right|_2$$

over the parameters a_i and $\dot{S}$ $(i = 1, \ldots, i_{max})$.

This leads to the set of MFE equations (ODE's)

$$A(\underline{y})\dot{\underline{y}} = g(\underline{y}).$$

where

$$\underline{y} = \{a_1, s_1 \ldots, a_N, S_N\}^T$$

$A(\underline{y})$ is the MFE matrix, which is square and symmetric, consisting of inner products of the basis function α and β in 2 x 2 blocks. $g(\underline{y})$ is a vector arising from $f(V)_x$.

The solution to this equation gives the solution of the grid points which vary with time and the solution V which corresponds to the grid point. The optimal feature of this method is that it allows the grid points to move to areas of the flow where large

gradients exist, thus giving high resolution in this area.

The model paths $x = s_i(t)$ are found to be approximations to the characteristic curves of (11), so that the nodal values $a_i(t)$ are almost independent of t. This observation has led Edwards[3] to propose a simpler version of MFE, which he calls the mobile element method (MEM). Here $s_i(t)$ is constrained to follow a characteristic path, and in (13) only the a_i have to be considered in the minimisation.

In practice we have not found either formulation to be satisfactory in the presence of a source term. Computations begin well, for a few time steps, but as nodes follow the characteristics they are removed from the reaction zone without being replaced. To date we have not succeeded in finding any theoretical framework that yields a satisfactory practical procedure.

All the other methods we use in this paper involve the concept of the Riemann problem, a description of which is now given.

3.3 Riemann Problem

The Riemann problem for a system (8), without the source terms, is the initial value problem for (8) with initial data

consisting of two constant states. Given the simplicity of the initial data one can solve this problem exactly if needed. The problem with more general data can then be reduced to sequences of Riemann problems by approximating the data by piece-wise constant functions.

We now present the results for the Riemann problem for the detonation analogue written as

$$U_t + A(U)U_x = 0 \tag{15}$$

where the Jacobian matrix A is given by

$$A(U) = \frac{\partial F}{\partial U} = \begin{bmatrix} \rho & \frac{1}{2}Q \\ 0 & 0 \end{bmatrix} \tag{16}$$

and whose eigenvalues are $e_1 = 0$ and $e_2 = \rho$ with corresponding right eigenvectors

$$V_1 = \begin{bmatrix} 1 \\ \frac{-2\rho}{Q} \end{bmatrix}, \quad V_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \tag{17}$$

The solution consists of two waves. The left wave $\frac{dx}{dt} = e_1 = 0$ is like a 'contact' across which ρ and λ change discontinuously and p is constant. The right wave $\frac{dx}{dt} = e_2 = \rho$ is either a shock or a rarefaction. The complete solution may be expressed as follows

$$\rho_m = [\rho_l^2 + Q(\lambda_l - \lambda_r)]^{\frac{1}{2}} \gtrless \rho_l \tag{18}$$

If $\rho_m > \rho_r$ then $p_m > p_r$ (shock). If $\rho_l < \rho_r$ then $p_m < p_r$ (rarefaction). Here subscript m denotes values in the uniform region between waves 1 and 2 and subscripts l and r denote left and right states (data for Riemann problem). More details about the solution of this problem can be found in Ref [4].

3.4 Godunov's Method

This two-step method was first proposed by Godunov [11]. It uses the solution of the Riemann problem to obtain provisional values for flux evaluation. The scheme is

$$U_i^{n+1} = U_i^n - \frac{\Delta T}{\Delta X}\left[F_{i+\frac{1}{2}}^{n+\frac{1}{2}} - F_{i-\frac{1}{2}}^{n+\frac{1}{2}}\right] \tag{19}$$

where $F_{i+\frac{1}{2}}^{n+\frac{1}{2}}$ is the flux F in system (8) evaluated at the solution of the Riemann problem with data U_i^n and U_{i+1}^n at $X = i\Delta X$ and $t = (n + \frac{1}{2})\Delta T$.

3.5 The Random Choice Method

This method was first presented as a computational technique by Chorin[5] in 1976. A number of improvements have since then been incorporated into the basic technique. An up-to-date account of the method as applied to the Euler equations in one dimension can be found in Ref. 6. Essentially, the Random Choice Method (RCM) solves the sequence of Riemann problems at a given time level n exactly. In a given computational cell i of size ΔX there will be waves from the left Riemann problem (i-1, i) and from the right Riemann problem (i, i+1). The solution at the

grid point i at time level n+1 will be taken as the solution of the Riemann problems concerned evaluated at a random position within cell i. A unique feature of this method is its ability to represent discontinuities with zero width.

3.6 A Higher Order Random Choice Method

Essentially, this method as presented in Ref 7 is a random generalisation of the Godunov's method, whereby intercell fluxes are evaluated at the solution of the corresponding Riemann problem at a random position as in the two-step version of the Random Choice Method. This technique turns out to be second order accurate. A hybridised version of this method was presented in Ref. 8 where traditional RCM is used at large discontinuities. These ideas are still in development, but our experience so far from applications to various problems is encouraging.

3.7 A Flux Difference Splitting Method

Several numerical schemes are based on an approximate wave analysis of the interaction between adjacent cells i,i+1; that is to say they solve an approximate Riemann problem for that data. Roe's method[9] is to solve the Riemann problem for a local linearisation of the governing equations

$$U_t + \tilde{A}(U_L, U_R)U_X = 0$$

where $\tilde{A}$ is a local average of the Jacobian matrix, chosen to have the property

$$\tilde{A}(U_L,U_R)(U_R - U_L) = F_R - F_L$$

This property ensures the solution is exact if U_R, U_L are states that can be connected by a single discontinuity. For the detonation analogue this matrix is very simple, it is

$$\tilde{A}(U_L,U_R) = \begin{bmatrix} \frac{1}{2}(\rho_L + \rho_R) & \frac{1}{2}Q \\ 0 & 0 \end{bmatrix} \qquad (20)$$

(Compare equation (16).

The eigenvalues and eigenvectors of this matrix are as given in Section (3.3), but with ρ replaced by $\frac{1}{2}(\rho_L + \rho_R)$.

To compute the interface flux by Roe's method, the first step is to express $U_R - U_L$ as

$$U_R - U_L = \sum_K \alpha_K V_K$$

where $\{V_K\}$ are the eigenvectors of $\tilde{A}$, and $\{\alpha_K\}$ are the coefficients of the expansion. Then the interface flux is given by

$$F_{i+\frac{1}{2}} = \frac{1}{2}(F_L + F_R) - \frac{1}{2} \sum_K \alpha_K |\lambda_K| V_K \qquad (21)$$

where L,R refer to i,i+1, and $|\lambda_K|$ is the absolute value of an eigenvalue of $\tilde{A}$.

This flux defines a robust first-order scheme, suitable for use in non-smooth regions of the flow, and having properties very similar to those of Godunov's scheme. In smooth regions it is allowable to add terms that produce second-order accuracy;

the flux equation (21) becomes

$$F_{i+\frac{1}{2}} = \tfrac{1}{2}(F_L + F_R) - \tfrac{1}{2}\sum_K \alpha_K |\lambda_K| [1 - \varphi_K(1 - |\nu_K|)]V_K \qquad (22)$$

where φ_K is a function equal to zero in non-smooth parts of the flow, but close to unity in smooth regions. Note that a separate smoothness monitor φ_K is used for each wave. In (22) ν_K is the Courant number associated with the Kth wave, that is

$$\nu_K = \frac{\lambda_K \Delta t}{\Delta X}$$

In smooth regions, putting $\Phi_K = 1$ yields

$$F_{i+\frac{1}{2}} = \tfrac{1}{2}(F_L + F_R) - \frac{\Delta t}{2\Delta t} \sum \alpha_K \lambda_K^2 V_K$$

$$= \tfrac{1}{2}(F_L + F_R) - \frac{A\Delta t}{2\Delta X}(F_R - F_L)$$

which recovers the flux of the Lax-Wendroff formula.

Different methods of computing φ_K are discussed in [10] and some of the references quoted there. In our present calculations we have used the "Superbee" limiter function.

4. TEST PROBLEMS

We consider two test cases, namely problems 1 and 2. For problem 1 the initial condition (t=0) is the steady exact solution; the boundary condition on the left boundary is $\rho_b = \rho_e$ and parameters are chosen so that $\rho_e = S$, where S is the steady shock speed. The computed results will show profiles of density, reaction progress variable and pressure for ten different times. Note that a reaction-zone width corresponds

unit time. The initial profile is also shown in the figures. This is the simplest test case one can devise for the analogue and yet it will show quite clearly what the limitations of some of the methods applied are. The exact solution is used for comparison with the numerical solution.

Problem 2 resembles more realistic detonation problems. The initial profile is a flat topped (unreactive) shock which initiates reaction. After a sufficiently long time (about five time units) a steady detonation has been developed. Parameters are chosen so that a steady solution as in problem 1 is obtained, ie. $\rho_b = \rho_e = S$.

In both problems 1 and 2 the choice on an 'ignition criterion' is necessary. For all the cases considered we took the value $\rho = 4.0$ as the value above which chemical reaction took place.

5. NUMERICAL RESULTS AND ASSESSMENT OF METHODS

Here we present one set of results for each of the two test cases specified in Section 4 (problems 1 and 2). All results are for a fixed and regular mesh. The chosen mesh size DX = 0.5 is such that the reaction zone is discretised by ten points; this is perhaps the maximum number that one could afford to use in realistic computations using regular fixed meshes.

5.1 Results For Problem 1

All computed results for this problem are shown in Figs. 2 to 7. The respective numerical techniques used are:

MacCormack's Method, the Moving Finite Element Method, Godunov's Method, the Random Choice Method, the Hybrid Method and the Flux Difference Splitting (Roe) Method. Values of some of the computing parameters used are shown on the figures. The displayed quantities are the density, the reaction progress variable and the pressure (computed from the equation of state). Eleven profiles are shown in each figure, except for the MacCormack's method, in which, to avoid confusion, only three profiles are shown. On each plot, the first profile on the left is the initial profile (time zero), which in this test case is the exact steady solution. All subsequent profiles come in pairs, showing the numerical solution (symbols plus dashed line) and the exact solution (full line). All methods were run up to about ten time units (about ten reaction zones).

The objective in mind for this test problem was to assess the performance of the various candidate numerical methods on the simplest problem, which in addition has an exact solution. Since the initial profile is the exact solution itself, the least that is expected from the numerical technique in use is preservation of the essential features of the solution (shock and the reaction zone), independently of the number of time steps used in the computations.

Fig. 2 shows results obtained using MacCormack; only two computed profiles are displayed. Quite clearly, these results are completely unacceptable. The typical overshoots and spurious

oscillations behind the shock are present (no artificial viscosity used). From the density (or pressure) profiles it is impossible to identify the main features of the problem. Note however that the solution for the reaction progress variable is slightly better in that it is possible to see where reaction has taken place. But since we are using an 'activation density' equal to 4.0, undershoots in the density profile below this value cause the reaction to stop temporarily. Otherwise the profile for λ should be smooth, although not necessarily correctly positioned. This is because the rate equation considered here is independent of density. Hence for more realistic rate equations the MacCormack solution for λ should be even worse that that of Fig. 2. We note that by using a mesh containing 50 points within the reaction zone it is possible to obtain a significantly better solution, although still unsatisfactory. But this sort of mesh is completely unrealistic.

Fig. 3 shows results obtained using the Moving Finite Element Method. The results are perfect for this problem.

Fig. 4 shows results obtained using Godunov's Method. Results are significantly better than those obtained using MacCormack, but the shock is smeared and the discontinuity in derivative at the end of the reaction zone is completely missed, as expected from a first order difference method. Consequently, the computed reaction zone has more than twice the correct width, after about ten time units. Also, shock

peak values are not attained accurately, the error is about 10%. This method, as it stands, is not a serious candidate for detonation problems.

Fig. 5 illustrates the results obtained by the Random Choice Method. These are very satisfactory. The shock is absolutely sharp (zero width), as guaranteed by RCM, although its position is affected by randomness. The discontinuity in derivative at the end of the reaction zone is very accurately captured. The reaction zone has the correct width, but the profile inside it is also affected by randomness (not the reaction progress variable). Shock peak values are very accurate. The disadvantage of the method is the randomness, which is more clearly manifested in the pressure profiles in the present computations.

Fig. 6 shows the results obtained by the Hybrid Method (Higher Order Random Choice plus RCM at large discontinuities). These results look very good. The typical randomness of RCM has been eliminated and the zero-width shock of RCM has been retained. The shock position, however, is still affected by certain randomness.

Fig. 7 illustrates the results obtained using Roe's method. They are very satisfactory. The shock is smeared but not as much as in the results obtained by first order Godunov's method (Fig. 4). Also, it is known that this high resolution scheme performs very well in more realistic gas dynamical problems,

as compared with other modern numerical methods.

5.2 Results for Problem 2

This test case was described in Section 4. The problem simulates detonation initiation by a flat-topped shock of sufficient strength. The intention is to assess the ability of the methods to 'grow' the detonation to the steady state solution. Results are shown in Figs. 8 to 11. The methods tested are Godunov, RCM, Hybrid and Roe's. We have excluded MacCormack from these results. Problem 1 has already shown its inadequacy. Also, we have excluded the Moving Finite Element Method. We could not succeed in producing acceptable solutions for this more realistic problem.

The performance of the various methods considered here is similar to that on Problem 1. They are all capable of initiating the detonation and carry it to steady state in about the same time (about five time units).

6. CONCLUSIONS

The numerical experiments using Problems 1 and 2 suggest that both MacCormack and Godunov methods are inadequate for the problem under study. Our MFE method could only succeed in the simplest problem although there the results were impressive. The remaining three methods (RCM, Hybrid and Roe) are presently being assessed in terms of (a) performance for the unsteady Euler equations in two space dimensions (b) ability of the method to incorporate complicated equations of state (c) viability

to implement the method using irregular, adaptive grids (d) computational efficiency and, naturally (e) degree of development of the technique and experience from applications to similar problems. Given these criteria, Roe's Method is a good candidate for the full physical problem, although we are still working on the key difficulty affecting Hybrid, and more particularly RCM; that is, the extension of this method to multi-dimensional problems preserving the quality of their one-dimensional performance.

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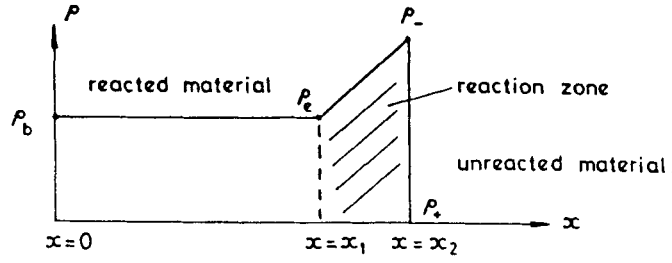
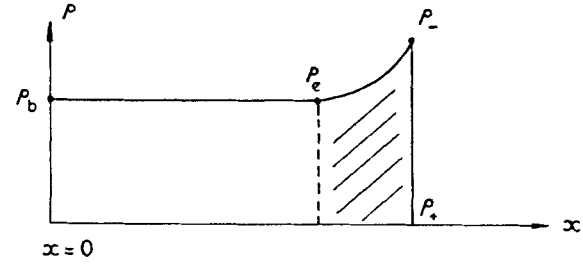
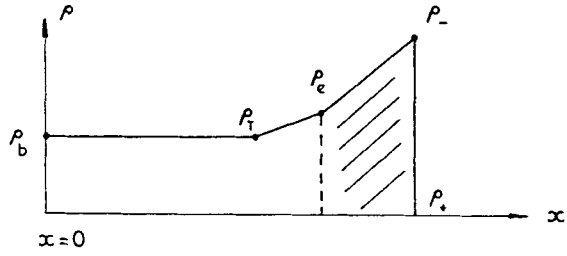


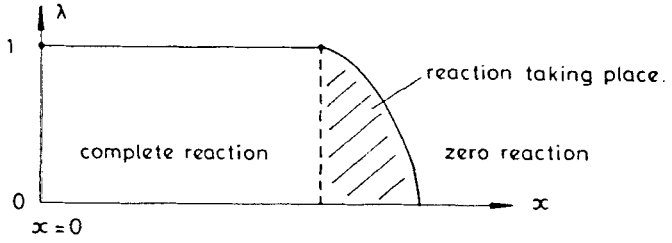
1(a) Overdriven Detonation, $P_b = P_e = S$



1(b) Overdriven Detonation, $P_b = P_e > S$

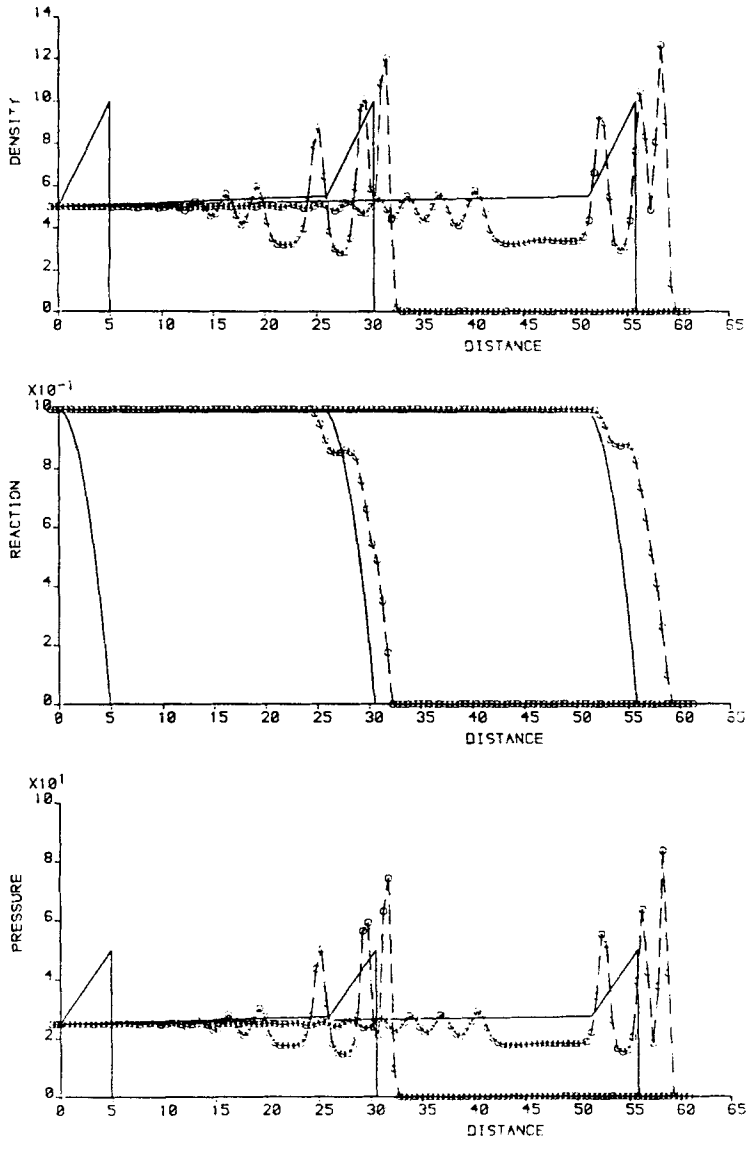


1(c) Unsupported Detonation, $P_y = P_T < P_e = S$

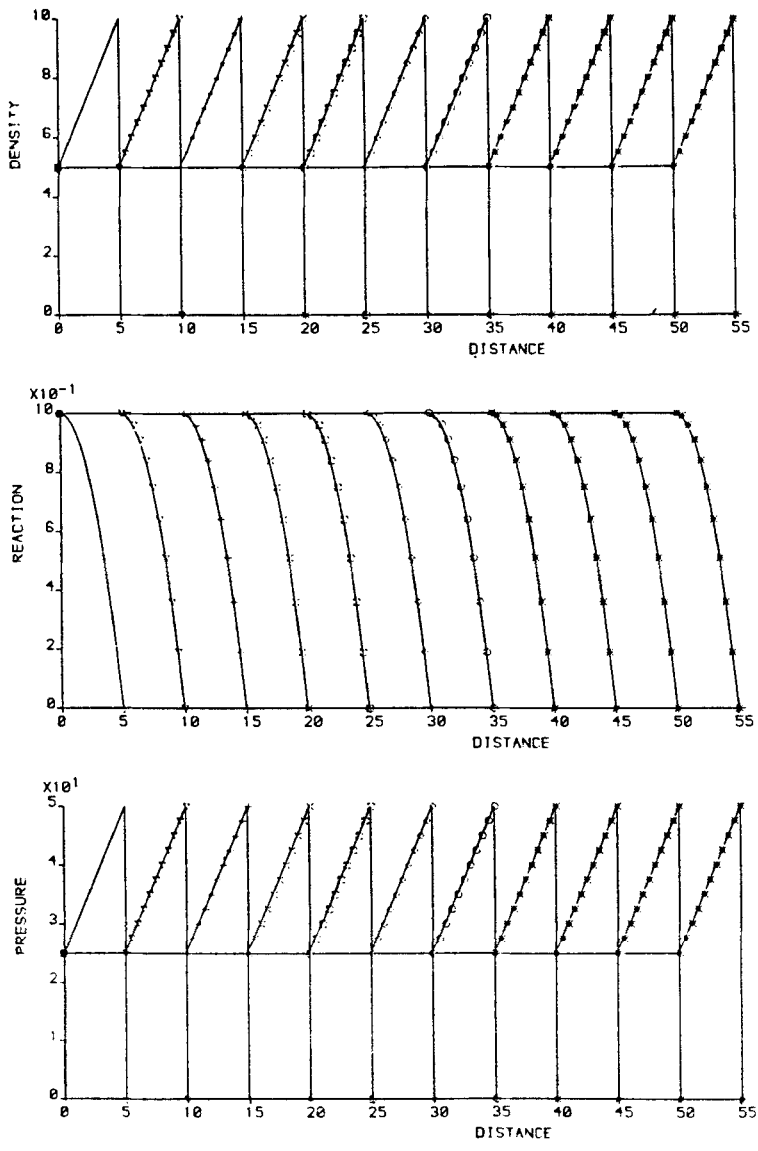


1(d) Reaction Progress Variable

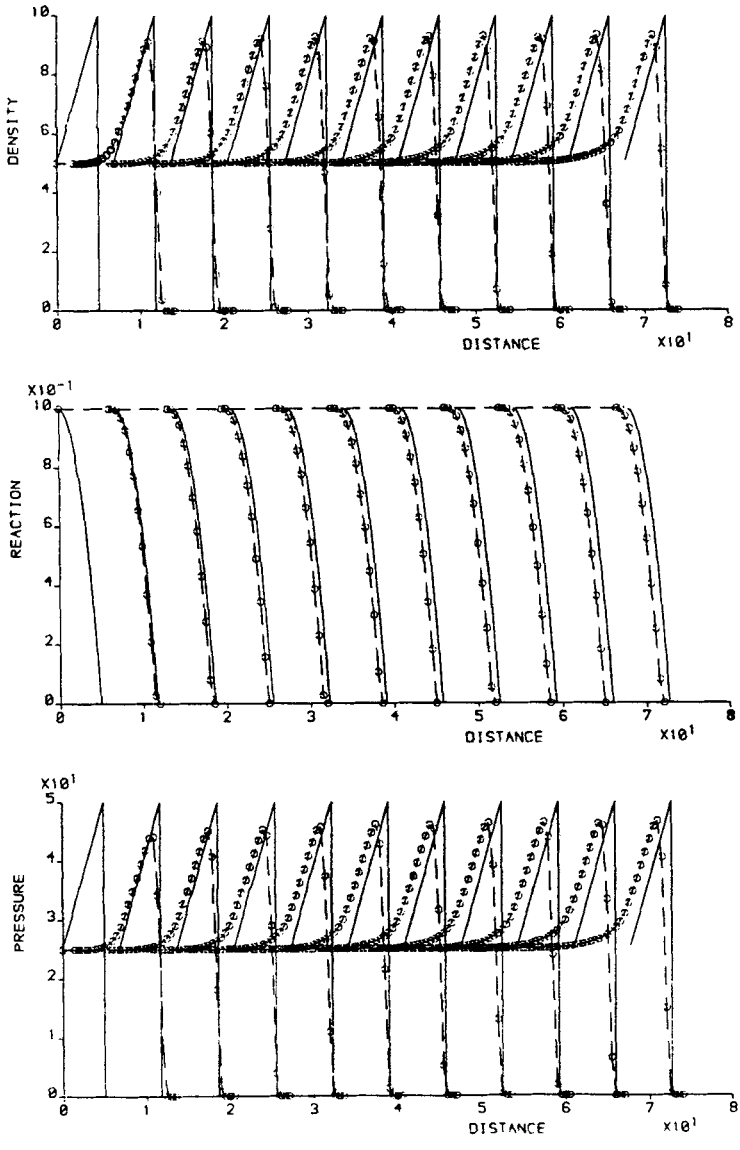
Fig 1: Likely solution profiles of anologue.



PROBLEM 1 (STEADY)
FIGURE 2 :NUMERICAL SOLUTION BY THE MACCORMACK METHOD AND EXACT SOLUTION (FULL LINE). Q= 25.00 S= 5.00 CFL NUMBER=0.25

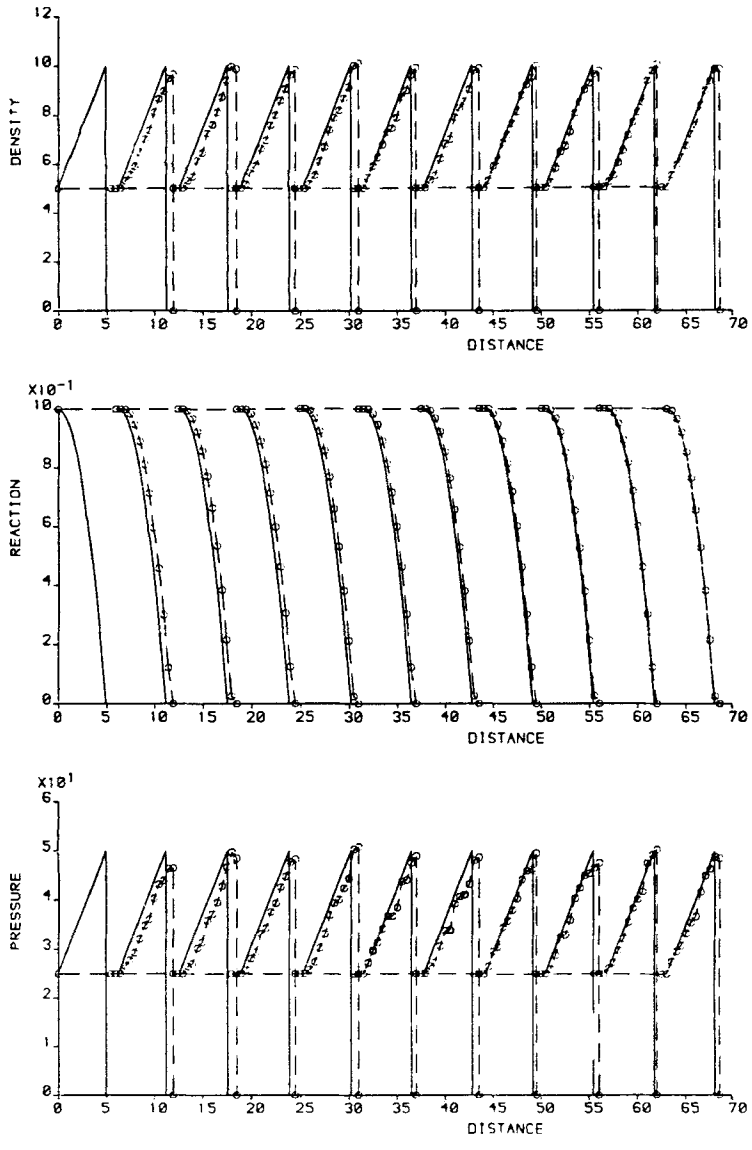


DETONATION ANALOGUE
FIGURE 3: MEM-COMPUTED (DOTTED) AND EXACT (FULL) SOLUTIONS.
Q= 25.00 S= 5.00 DT= 0.0050 NO. OF NODES= 10 TFINAL= 10.00

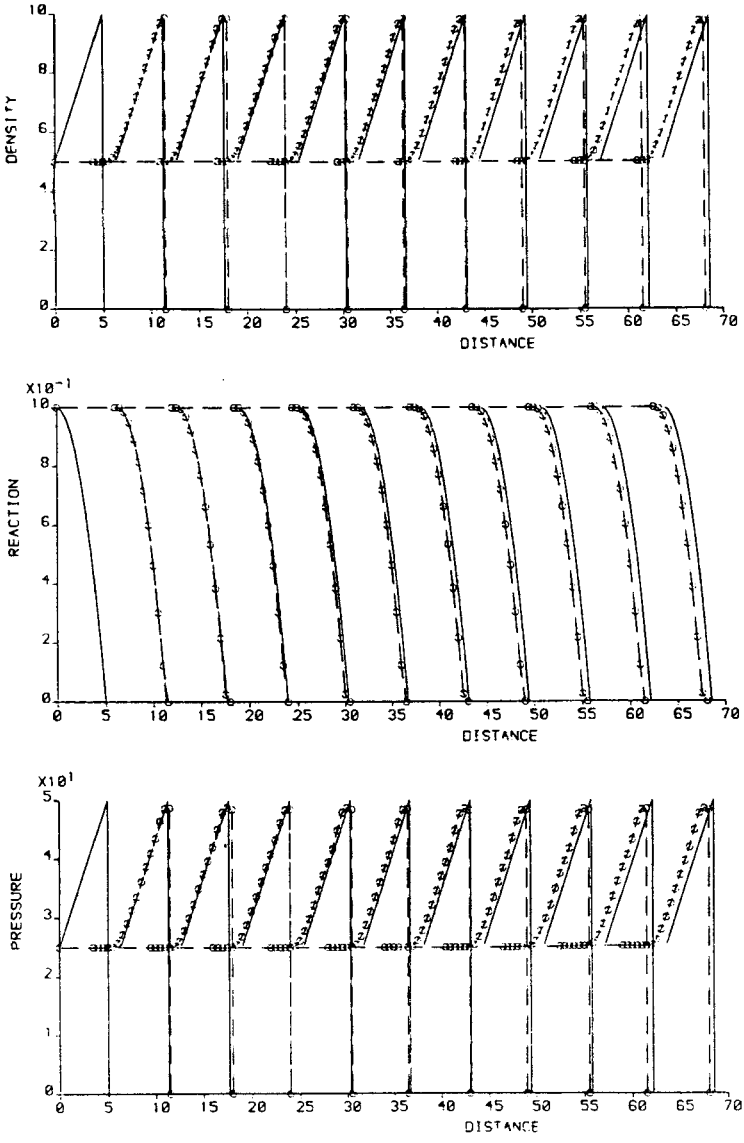


PROBLEM 1 (STEADY)

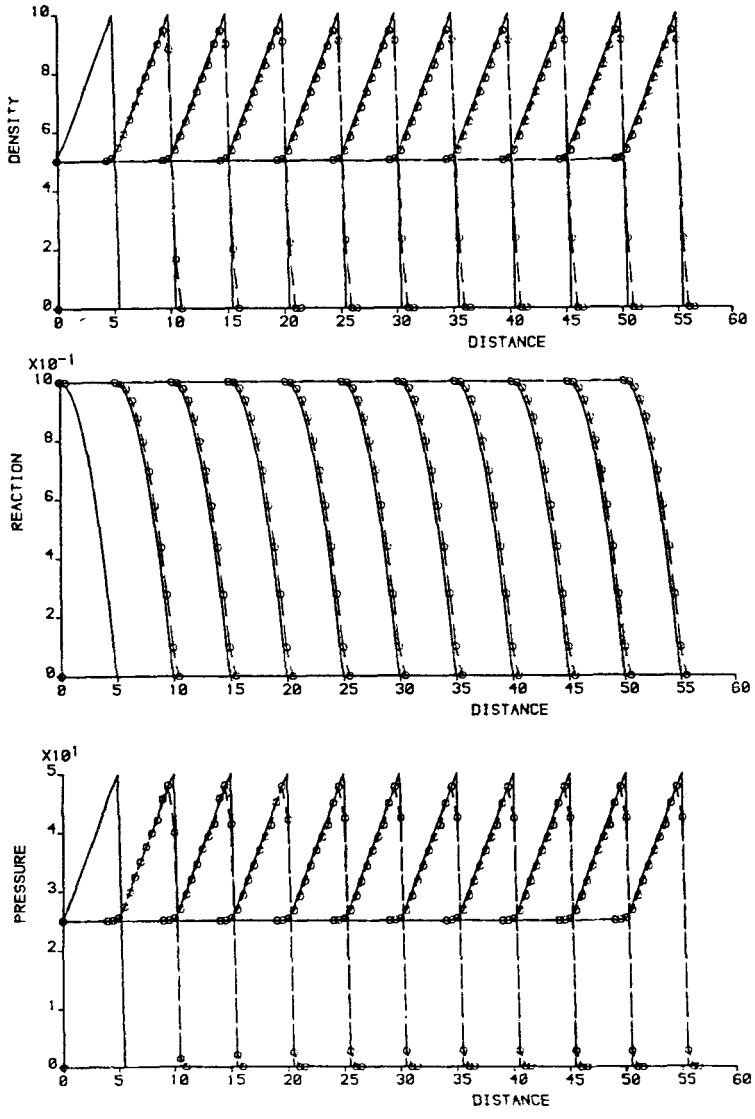
FIGURE 4 :NUMERICAL SOLUTION USING THE GODUNOV'S METHOD AND EXACT SOLUTION (FULL LINE). Q= 25.00 S= 5.00 CFL NUMBER=0.50



PROBLEM 1 (STEADY)
FIGURE 5 :NUMERICAL SOLUTION BY RANDOM CHOICE METHOD AND EXACT SOLUTION (FULL LINE). Q= 25.00 S= 5.00 CFL NUMBER=0.50



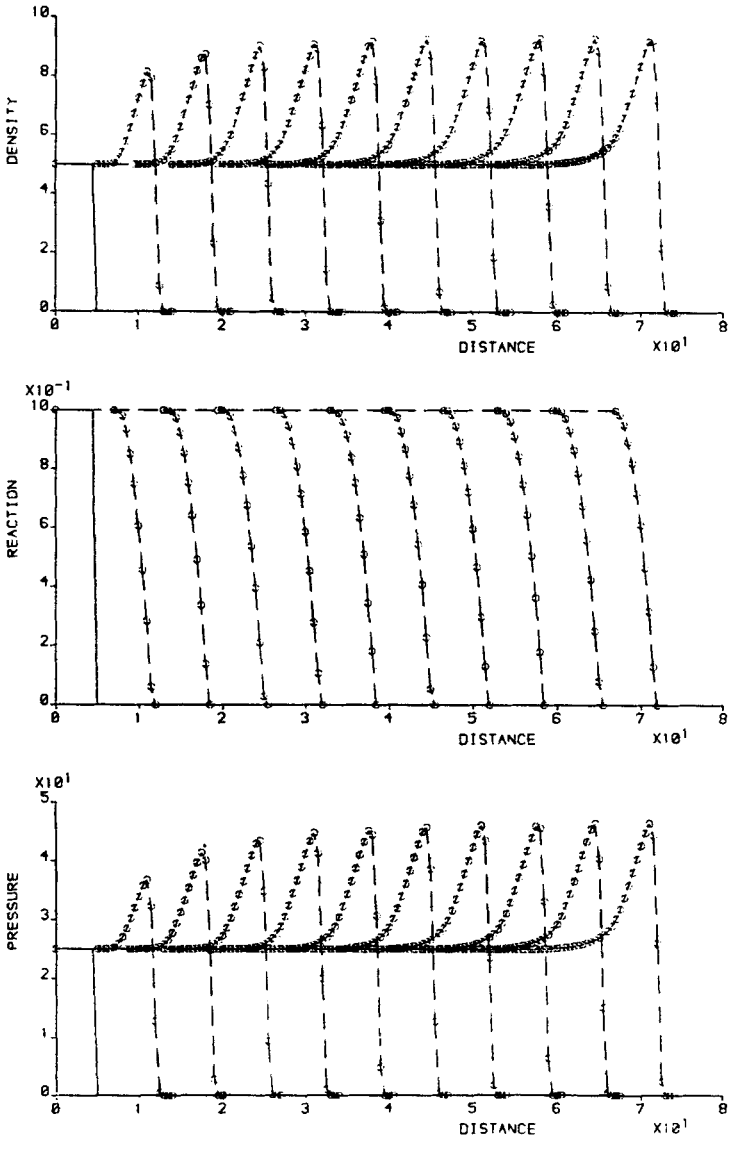
PROBLEM 1 (STEADY)
FIGURE 6 :NUMERICAL SOLUTION USING THE HYBRID METHOD AND EXACT SOLUTION (FULL LINE). Q= 25.00 S= 5.00 CFL NUMBER=0.50



PROBLEM 1 (STEADY)

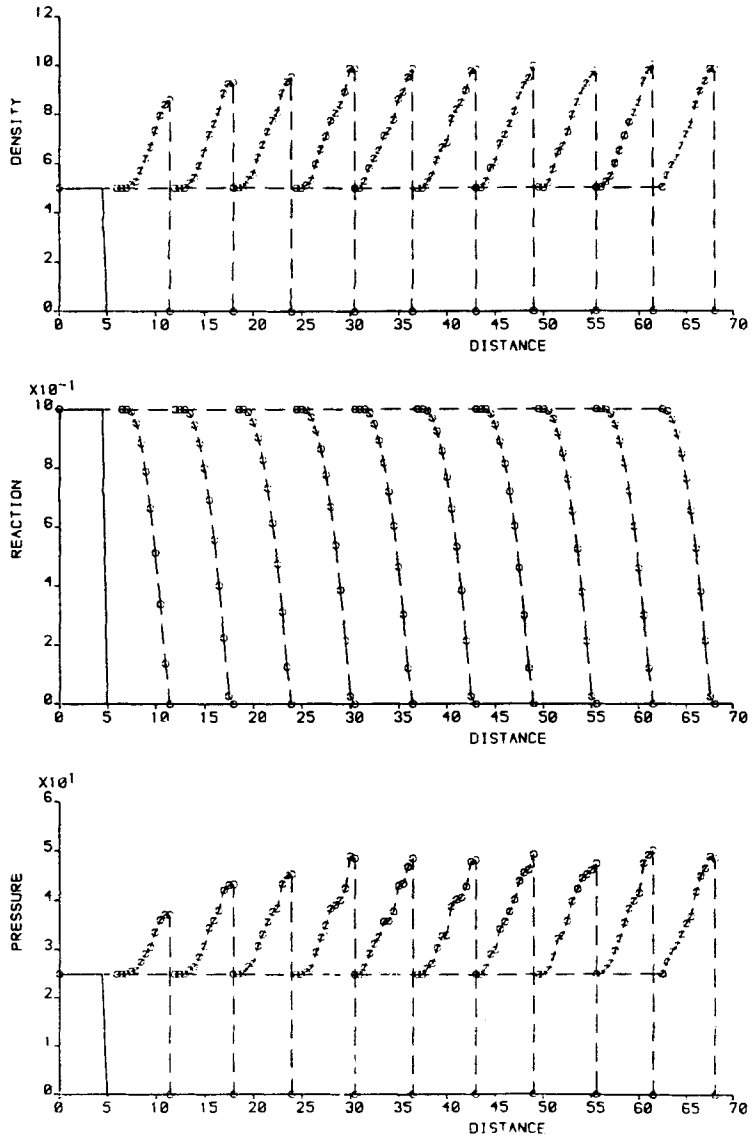
FIGURE 7 : NUMERICAL SOLUTION BY ROE'S METHOD AND EXACT SOLUTION (FULL LINE) ON REGULAR GRID

Q= 25.00, S=5.00, DT=0.025, DX= 0.50, CFL=0.500 IMAX=115 NTS=400

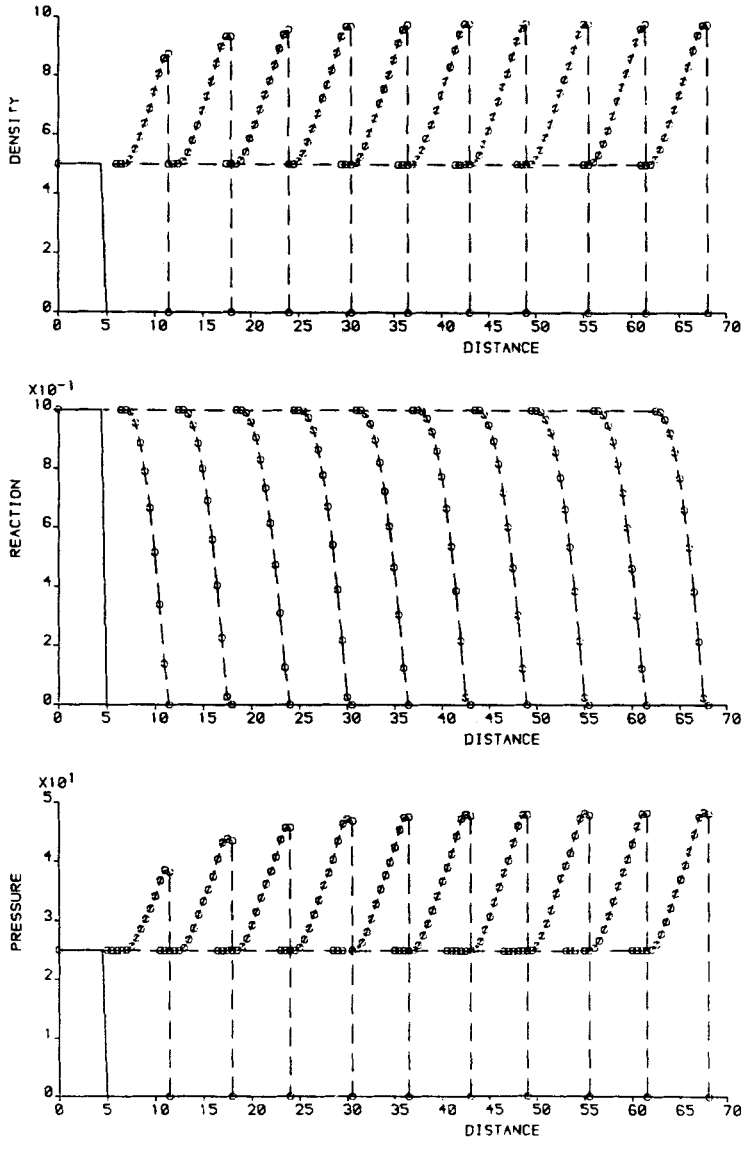


PROBLEM 2 (UNSTEADY)

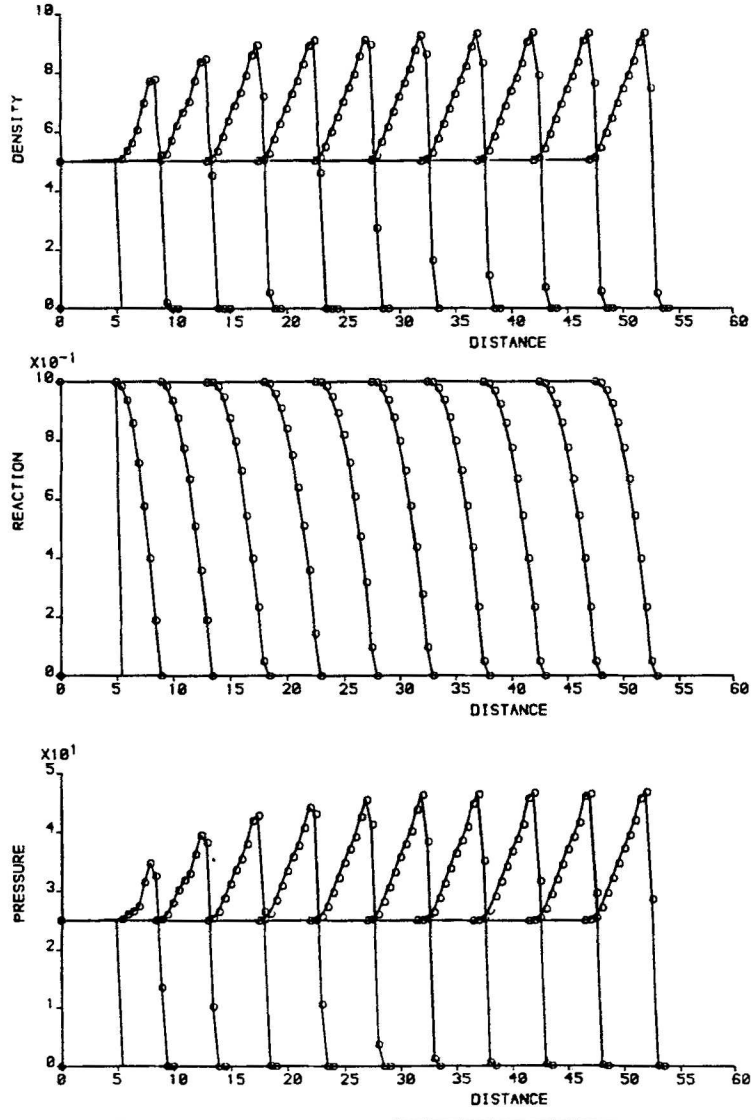
FIGURE 8 :NUMERICAL SOLUTION USING GODUNOV'S METHOD.
Q= 25.00 S= 5.00 CFL NUMBER=0.50



PROBLEM 2 (UNSTEADY)
FIGURE 9 :NUMERICAL SOLUTION BY RANDOM CHOICE METHOD.
Q= 25.00 S= 5.00 CFL NUMBER=0.50



PROBLEM 2 (UNSTEADY)
FIGURE 10 :NUMERICAL SOLUTION USING THE HYBRID METHOD.
Q= 25.00 S= 5.00 CFL NUMBER=0.50



PROBLEM 2 (UNSTEADY)

FIGURE 11: NUMERICAL SOLUTION BY ROE'S METHOD ON REGULAR GRID

Q= 25.00, S=5.00, DT=0.025, DX= 0.50, CFL=0.500 IMAX=112 NTS=400